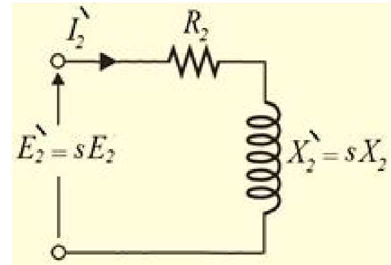
Torque of Induction Motor under Running Condition (T_r)



- It is the torque developed by the motor under running condition (i.e., at slip *s*). From figure, the rotor circuit/phase at slip *s*.
- Let,

$E_2^{\backslash} = sE_2$: rotor e.m.f/phase under running condition (*volts*)

$X_2^{\backslash} = sX_2$: rotor reactance/phase under running condition (Ω)

R_2 : rotor resistance/phase under running condition (Ω)

$Z_2^{\backslash} = \sqrt{R_2^2 + (sX_2^{\backslash})^2}$: rotor impedance/phase under running condition (Ω)

Rotor current/phase, $$I_2^{\backslash} = \frac{E_2^{\backslash}}{Z_2^{\backslash}} = \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}}$$ … under running conditions

Rotor *p.f*, $$\cos\phi_2^{\backslash} = \frac{R_2}{Z_2^{\backslash}} = \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}}$$ … under running conditions

Torque of Induction Motor under Running Condition (T_r) …

- Since, the motor torque/phase at slip *s* (or under running conditions) is:

$$T_r \ \alpha \ \Phi \ I_2^{\backslash} \cos\phi_2^{\backslash}$$

$$\alpha \ \Phi \ \frac{E_2^{\backslash}}{Z_2^{\backslash}} \frac{R_2}{Z_2^{\backslash}}$$

$$\alpha \ \Phi \ \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}} \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}} \qquad \because E_2 \ \alpha \ \Phi$$

$$\alpha \ E_2 \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}} \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

$$T_r = \frac{K\, sE_2^2 R_2}{R_2^2 + (sX_2)^2} = \frac{K\, sE_2^2 R_2}{Z_2^{\backslash 2}} \qquad \underline{N_s \ is \ synch.\ speed \ (rps)}$$

$$if \ K = \frac{1}{2\pi N_s} \rightarrow T_r = \frac{1}{2\pi N_s} \cdot \frac{sE_2^2 R_2}{R_2^2 + (sX_2)^2} \ .... torque \ per \ phase$$

$$if \ K = \frac{3}{2\pi N_s} \rightarrow T_r = \frac{3}{2\pi N_s} \cdot \frac{sE_2^2 R_2}{R_2^2 + (sX_2)^2} \ .... motor \ torque \ under \ running \ condition$$

Torque of Induction Motor under Running Condition (T_r) …

- If the stator (or supply) voltage V is constant, then stator flux and hence E_2 will be *constants*.

$$T_r = \frac{K\,sE_2^2\,R_2}{R_2^2 + (sX_2)^2} = \frac{K_1\,sR_2}{R_2^2 + (sX_2)^2}$$

Where: the constant K not equal to K_1

$$K_1 = K\,E_2^2$$

- It may be seen that the running torque is:

i- Directly proportional to the slip. If the slip increases (*i.e.*, motor speed decreases), the torque will increase and vice-versa.

ii- Directly proportional to square of supply voltage, because $\because E_2 \,\alpha\, V$

Example 5: A Y-connected rotor of an induction motor has standstill impedance of (0.4+j4) ohm per phase and the rheostat impedance per phase is (6+j2) ohm. The motor has an induced *emf* of 80V between slip-rings at standstill when connected to its normal supply voltage. Find:

(i) rotor current at standstill when the rheostat is in the circuit.

(ii) rotor current at slip 3% when the slip-rings are short-circuited.

Answer

[Ans.: 5.265A - 3.32A]

Maximum Torque under Running Conditions ($T_{r(max)}$)

- Assuming the supply voltage is constant, to get the maximum value of the running torque, take a derivative for it w.r.t the constant R_2 and equating the result with zero as follows:

$$\because T_r = \frac{K_1 s R_2}{R_2^2 + (sX_2)^2}$$

$$\therefore \frac{dT_r}{dR_2} = K_1\left[\frac{(s)\times(R_2^2 + (sX_2)^2) - (2R_2)\times(sR_2)}{(R_2^2 + (sX_2)^2)^2}\right] = 0$$

$$\therefore sR_2^2 + s(sX_2)^2 - 2sR_2^2 = 0$$

$$\therefore R_2^2 = (sX_2)^2$$

$$\therefore R_2 = sX_2 \qquad i.e.,\ s = \frac{R_2}{X_2}\ under\ running\ condition$$

Hence, the running torque will be maximum when:

*Rotor resistance/phase = slip * Standstill rotor reactance/phase*

$$T_{r\,(\max)} = \frac{K_1 s}{2R_2} = \frac{K_1}{2X_2}$$

Maximum Torque under Running Conditions ($T_{r(max)}$)

- Also:

$$T_{r(\max)} = \frac{K_1}{2X_2} = \frac{KE_2^2}{2X_2} = \frac{3}{2\pi N_s}\frac{E_2^2}{2X_2} \qquad Max.\,motor\ torque\ under\ running\ condition$$

Notes:

- The maximum running torque is independent on the rotor resistance (because the rotor resistance is constant for the motor).
- The maximum running torque varies inversely with the standstill reactance. Hence, it should be kept as small as possible.
- The maximum running torque varies directly with the square of the applied voltage.

Example 6: 3-ph, slip-ring, induction motor with Y-connected rotor has an induced *emf* of 120V between slip-rings at standstill with normal voltage applied to the stator. The rotor winding has a resistance per phase 0.3 ohm and standstill leakage reactance per phase is 1.5 ohm. calculate:

(i) rotor current/phase when running short-circuited with 4% slip.

(ii) Slip and rotor current/phase when the rotor is developing maximum torque.

Answer

[Ans.: 9A - 0.2 - 33A]

Example 7: A 3-ph, induction motor having 6-pole, Y-connected stator winding runs on 240V, 50Hz supply. The rotor is connected as Y-connection. The rotor resistance and standstill reactance are 0.12 ohm and 0.85 ohm per phase. The ratio of stator to rotor turns is 1.8. Full-load slip is 4%. calculate:

(i) Developed torque at full-load.

(ii) Maximum torque and speed at that maximum torque.

Answer

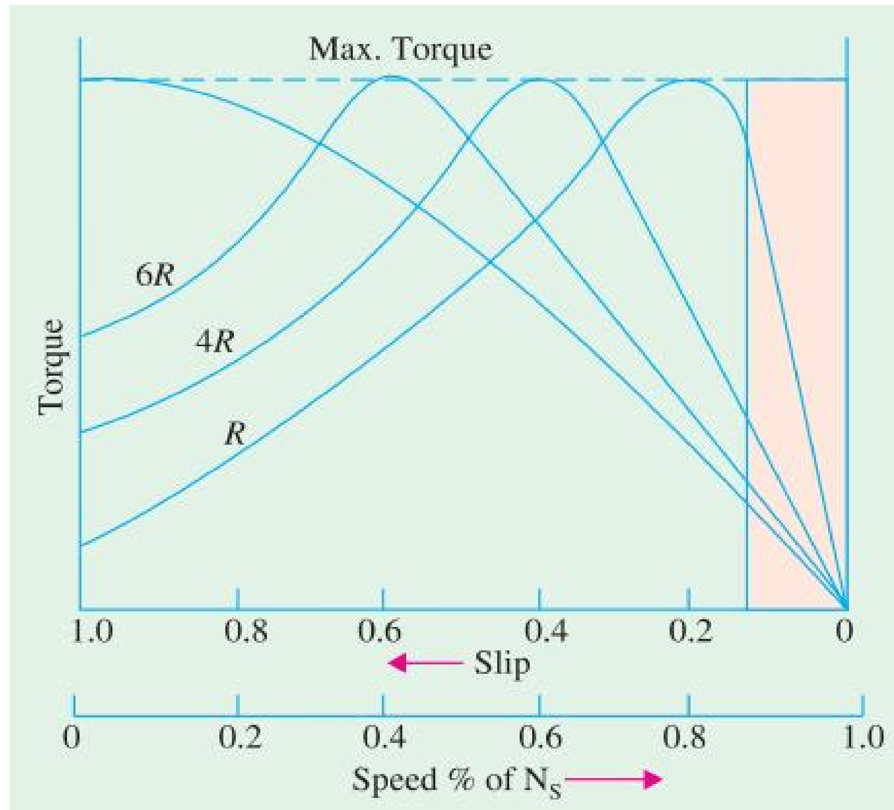
[Ans.: 52.4Nm - 99Nm - 860rpm]

Torque-Slip Characteristics

- The motor torque under running conditions is given by:

$$\because T_r = \frac{K_1 s R_2}{R_2^2 + (sX_2)^2}$$

If a curve is drawn between the torque and slip for a particular value of rotor resistance R_2; the graph thus obtained is called *torque-slip characteristic*. Fig. shows a family of torque-slip characteristics for a slip-range from s = 0 to s = 1 for various values of rotor resistance.

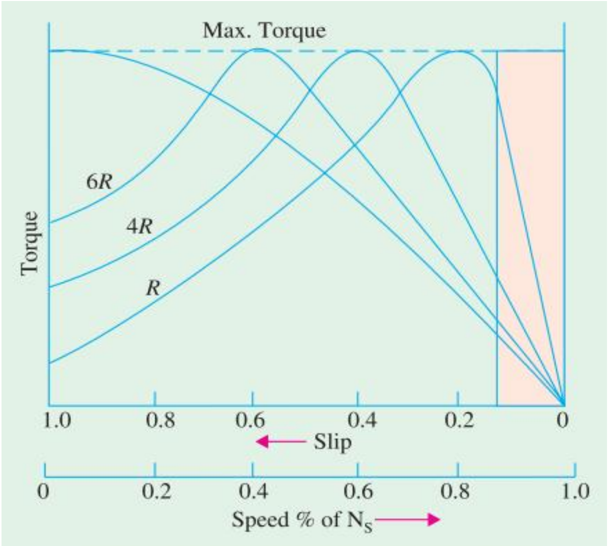


s =1 ... this is starting torque condition

Torque-Slip Characteristics …

The following points may be noted carefully:

$$\because T_r = \frac{K_1 s R_2}{R_2^2 + (sX_2)^2}$$



s=1 ... this is starting torque condition

(i) At $s = 0$, $T_r = 0$ so that torque-slip curve starts from the origin.

(ii) As slip increses beyond zero value (*i.e.*, slip is small but larger than zero… the case of normal speed that close to synchronism) so that sX_2 is negligible as compared to R_2.

$$\therefore T_r \ \alpha \ \frac{s}{R_2} \quad \rightarrow \quad \therefore T_r \ \alpha \ s$$

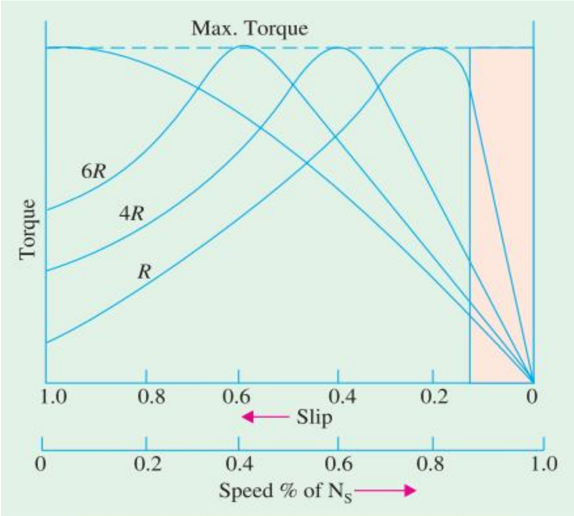
… linear relation between T_r & s

Hence torque-slip curve is *a straight line* from zero slip to a slip that corresponds to full-load (the shaded region at using the surve of R_2)

Torque-Slip Characteristics …

$$\because T_r = \frac{K_1 s R_2}{R_2^2 + (sX_2)^2}$$

(iii) As slip increases beyond full-load slip (for increasing load of the motor), the torque increases and becomes maximum at s = R_2/X_2. This maximum torque in an induction motor is called *pull-out* or breakdown torque. Its value is at least twice the full-load value when the motor is operated at rated voltage and frequency.



s = 1 ... this is starting torque condition

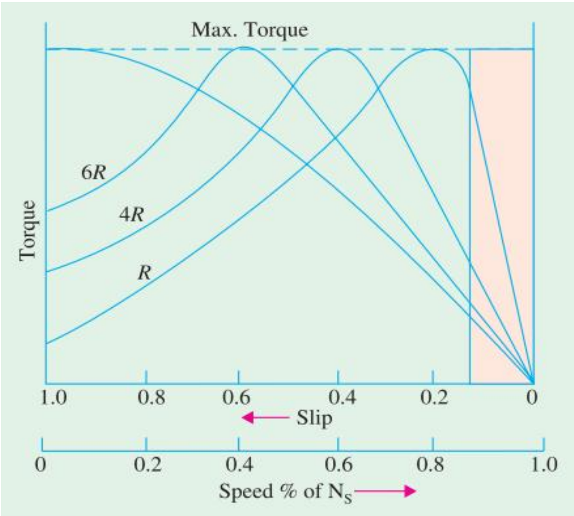
(iv) As the slip further increses (i.e., motor speed falls) with further increase in motor load, then R_2 becomes neglegible compared to sX_2. Therfore, the motor slows down and eventually stops.

$$\therefore T_r \ \alpha \ \frac{s}{(sX_2)^2} \quad \rightarrow \quad \therefore T_r \ \alpha \ \frac{1}{s}$$

In fact, the stable operation of the motor lies between the values of s = 0 and that corresponding to maximum torque.(is operating range shown shaded in the figure)

Torque-Slip Characteristics …

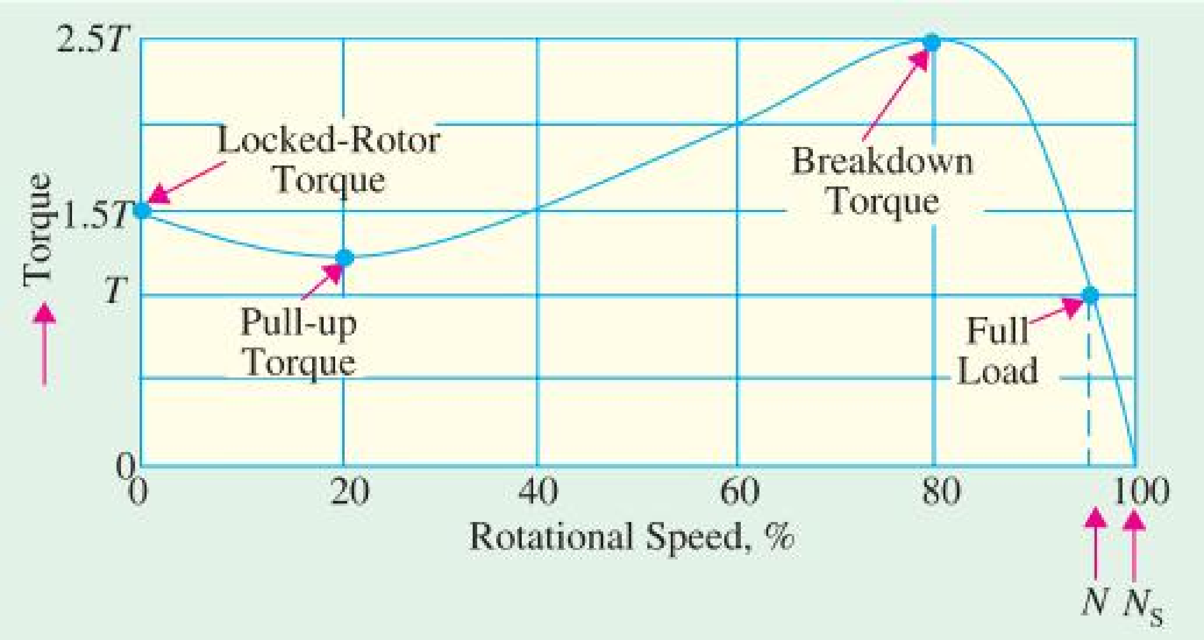
(v) Because the maximum running torque is independent of the value of rotor resistance, so, it is remains the same. Therefore, the addition of resistance to the rotor circuit does not change the value of maximum running torque but it is only changes the value of slip at which maximum torque occurs.



$s = 1$ ... this is starting torque condition

Torque-Speed Characteristics

The torque developed by a 3-ph induction motor depends on its speed but the relation cannot be represented by a simple equation. As shown in figure,



- T : represents nominal full-load torque at full-load speed N.
- $1.5T$: represents starting (or locked rotor) torque at ($N = 0$ … at $s=1$).
- $2.5T$: represents the maximum (or breakdown) torque at ($s = R_2/X_2$).
- If the load torque exceeds breakdown torque, the motor will suddenly stop.

Speed Regulation of Induction Motors

- It is the difference between no-load speed and full-load speed. It is an indication for motor performance, as the value of speed regulation decreases, the motor will have a bad performance

$$\%Speed\ regulation = \frac{N_{nl} - N_{fl}}{N_{fl}} \times 100$$

Where;

N_{nl} : no-load speed of the motor.

N_{fl} : full-load speed of the motor.

▪ If the no-load speed of the motor is 800r.p.m. and its full-load speed is 780r.p.m, then change in speed is 800−780=20r.p.m. and %speed regulation =20×100/780 = 2.56%.

▪ The speed regulation of an induction motor is 3% to 5%. This value is low enough. So, the induction motor is classed as *a constant-speed motor*.

Speed Control of 3-Phase Induction Motors

$$N = (1 - s) N_s = (1 - s) \frac{120 f}{P}$$

- The speed of induction motor (N) can be varied by changing:
  - (i) Supply frequency, f.
  - (ii) No. of stator poles, P.
  - (iii) The slip, s.

The change of f is generally not possible because the commercial supplies have constant frequency. Therefore, the practical methods of speed control are either to change P or s.

Speed Control of 3-Phase Induction Motors …

1. Squirrel Cage Motors

Its speed changed by changing number of stator poles (only two or four speeds are possible by this method). Two-speed motor has one stator winding provides two speeds (For instance, the winding may be connected for either 4 or 8 poles, giving N_s of 1500 or 750r.p.m. Four-speed motors are equipped with two separate stator windings each of which provides two speeds.

2. Wound rotor motors

The speed of wound rotor motors is changed by changing the motor slip. This can be achieved by;

- Varying the stator line voltage.
- Varying the resistance of the rotor circuit.